

Non-anomalous Discrete R -symmetry, Extra Matters, and Enhancement of the Lightest SUSY Higgs Mass

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Abstract

We consider low-energy supersymmetric model with non-anomalous discrete R -symmetry. In such a model, to make the R -symmetry non-anomalous, new particles with gauge quantum numbers should be inevitably added to the particle content of the minimal supersymmetric standard model (MSSM). Those new particles may couple to the Higgs boson, resulting in a significant enhancement of the lightest Higgs mass. We show that, in such a model, the lightest Higgs mass can be much larger than the MSSM upper bound; the lightest Higgs mass as large as 140 GeV (or larger) becomes possible.

1 Introduction

If supersymmetry (SUSY) survives at low-energy scale (~ 1 TeV), discrete R -symmetries Z_{NR} (with $N > 2$) seem to play an important role. First of all, the constant term in superpotential, that is the gravitino mass, is only controlled by the discrete R -symmetries. Therefore, the low-scale breaking of the discrete R -symmetries may account for the presence of SUSY at TeV scale, for example. The discrete R -symmetries may play a role of suppressing dangerous dimension 4 operators for proton decays and they also may guarantee the required long lifetime of the lightest SUSY particle as a dark matter candidate.^{#1} It is known that approximate continuous R -symmetries seem an essential component of dynamical SUSY breaking. Those approximate $U(1)_R$ symmetries might be realized as an accidental effective symmetry of the discrete R -symmetries Z_{NR} with $N > 2$.

However, the discrete Z_{NR} symmetries have gauge anomalies in the minimal SUSY standard model (MSSM) [2]. Thus, if it is the case, there is no reason to assume such discrete symmetries as an (almost) exact symmetries. Therefore it seems more interesting to cancel the unwanted anomalies by adding extra matters in the MSSM. In [3] it is shown that the discrete Z_{4R} symmetry is one of a few candidates. In fact, all gauge anomalies of Z_{4R} can be canceled out by adding a pair of $\mathbf{5} + \bar{\mathbf{5}}$ chiral multiplets or a pair of $\mathbf{10} + \bar{\mathbf{10}}$. Their masses are predicted to be at the same order of the Higgsino mass, say ~ 1 TeV. Thus, they must give a significant contribution to low-energy physics. In particular, in the latter solution, up-type Higgs H_u can couple to the extra matters in the $\mathbf{10}$ multiplet as $W \sim UQH_u$ (where U and Q have the same gauge quantum numbers as right-handed up-type quarks and left-handed quark doublets, respectively).

In this paper, we show that the mass of the lightest Higgs boson can be raised up to ~ 140 GeV because of the extra Yukawa coupling even when the SUSY-breaking scale is 1 TeV. This will be tested at LHC soon.

2 Non-Anomalous Discrete R -symmetry

In this section, following [3], we discuss anomaly-free conditions of discrete R -symmetry, Z_{NR} , in the framework of $SU(5)$ Grand Unified Theories (GUTs). We assume that neutrino masses are explained by seesaw mechanism [4, 5, 6]. Then, the superpotential of the minimal SUSY $SU(5)$ GUT is of the following form:

$$W_{\text{GUT}} \sim \Phi_{\mathbf{10}}\Phi_{\mathbf{10}}H + \Phi_{\mathbf{10}}\bar{\Phi}_{\bar{\mathbf{5}}}\bar{H} + \bar{\Phi}_{\bar{\mathbf{5}}}\bar{N}\bar{H} + \frac{1}{2}M_N\bar{N}\bar{N} + \mu_H H\bar{H}, \quad (1)$$

where M_N is the Majorana mass of right-handed neutrinos. The quantum numbers of the fields are shown in Table 1.

^{#1}When the discrete Z_{NR} (with $N > 2$) is broken down to the R -parity Z_{2R} , we may have too many domain walls. For solutions to this domain wall problem, see [1].

	$\Phi_{\mathbf{10}}$	$\bar{\Phi}_{\bar{\mathbf{5}}}$	\bar{N}	H	\bar{H}
$SU(5)_{\text{GUT}}$	$\mathbf{10}$	$\bar{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{5}$	$\bar{\mathbf{5}}$
Z_{NR}	$\phi_{\mathbf{10}}$	$\bar{\phi}_{\bar{\mathbf{5}}}$	$\bar{\nu}$	h	\bar{h}

Table 1: The matter content of the supersymmetric $SU(5)$ GUT, and the quantum numbers of the fields. The Z_{NR} charge of the Grassmann coordinate, θ , is denoted as α .

To make the model invariant under the discrete R -symmetry, the Z_{NR} charges should satisfy the following conditions:

$$2\phi_{\mathbf{10}} + h = 2\alpha \pmod{N}, \quad (2)$$

$$\phi_{\mathbf{10}} + \bar{\phi}_{\bar{\mathbf{5}}} + \bar{h} = 2\alpha \pmod{N}, \quad (3)$$

$$\bar{\phi}_{\bar{\mathbf{5}}} + \bar{\nu} + h = 2\alpha \pmod{N}, \quad (4)$$

$$2\bar{\nu} = 2\alpha \pmod{N}. \quad (5)$$

Here, we assume that the μ_H -term is generated by the Giudice-Masiero mechanism [7], and that Z_{NR} -symmetry prevents μ_H -parameter from being Planck scale. Then, the following conditions are imposed:

$$h + \bar{h} = 0 \pmod{N}, \quad \text{and} \quad h + \bar{h} \neq 2\alpha \pmod{N}, \quad (6)$$

which reduce to

$$2\alpha \neq 0 \pmod{N}. \quad (7)$$

Next, we consider the conditions for anomaly cancellation [2]. For $Z_{NR}[SU(3)_C]^2$ and $Z_{NR}[SU(2)_L]^2$:

$$Z_{NR}[SU(3)_C]^2 : \frac{3}{2} \{3(\phi_{\mathbf{10}} - \alpha) + (\bar{\phi}_{\bar{\mathbf{5}}} - \alpha)\} + 3\alpha = \frac{N}{2}k, \quad (8)$$

$$Z_{NR}[SU(2)_L]^2 : \frac{3}{2} \{3(\phi_{\mathbf{10}} - \alpha) + (\bar{\phi}_{\bar{\mathbf{5}}} - \alpha)\} + \frac{1}{2} \{(h - \alpha) + (\bar{h} - \alpha)\} + 2\alpha = \frac{N}{2}k', \quad (9)$$

where k and k' are integers. Using Eqs. (3), (4) and (6), these condition are rewritten as

$$Z_{NR}[SU(3)_C]^2 : 3\alpha = \frac{N}{2}k, \quad (10)$$

$$Z_{NR}[SU(2)_L]^2 : \alpha = \frac{N}{2}k'. \quad (11)$$

Importantly, Eq. (11) contradicts with the condition (7). Thus, an additional contribution from extra matters is needed to realize a non-anomalous discrete R -symmetry in the framework of $SU(5)$ GUT.

Because the extra matters have gauge quantum numbers, they have to be heavy enough to avoid direct search constraints. If the extra matters are vector-like, their masses can be generated by the Giudice-Masiero mechanism. If so, their masses are expected to be around the mass scale of MSSM superparticles (i.e., ~ 1 TeV). In order for the Giudice-Masiero mechanism to work, the following condition should be satisfied:^{#2}

$$\phi' + \bar{\phi}' = 0 \pmod{N}, \quad (12)$$

where ϕ' and $\bar{\phi}'$ are the Z_{NR} charge of the extra matter multiplets Φ' and $\bar{\Phi}'$, respectively.

Extra matters should be embedded in complete multiplets of the GUT group $SU(5)_{\text{GUT}}$ in order not to spoil the gauge coupling unification. In addition, requiring the perturbativity of the gauge coupling constants up to the GUT scale, we cannot introduce too many extra matters. If the masses of extra matters are at ~ 100 GeV – 1 TeV, only a limited number of $\mathbf{5} + \bar{\mathbf{5}}$ and/or $\mathbf{10} + \bar{\mathbf{10}}$ pairs can be introduced; if a larger representation of $SU(5)_{\text{GUT}}$ is added at $\mu \lesssim 1$ TeV, the gauge couplings become non-perturbative below the GUT scale. Denoting the numbers of $\mathbf{5} + \bar{\mathbf{5}}$ and $\mathbf{10} + \bar{\mathbf{10}}$ pairs as $n_{\mathbf{5}'}$ and $n_{\mathbf{10}'}$, respectively, perturbativity of the gauge couplings requires

$$n_{\mathbf{5}'} + 3n_{\mathbf{10}'} \leq 4. \quad (13)$$

In addition, using Eq. (12), the conditions for the anomaly cancellation are

$$Z_{NR}[SU(3)_C]^2 : (3 - n_{\mathbf{5}'} - 3n_{\mathbf{10}'})\alpha = \frac{N}{2}k, \quad (14)$$

$$Z_{NR}[SU(2)_L]^2 : (1 - n_{\mathbf{5}'} - 3n_{\mathbf{10}'})\alpha = \frac{N}{2}k'. \quad (15)$$

The conditions (7), (13), (14) and (15) are simultaneously satisfied only when $(n_{\mathbf{5}'}, n_{\mathbf{10}'}) = (1, 0)$, $(3, 0)$, or $(0, 1)$. In these cases, N should be 4 or 20 [3]; for $N = 4$ and 20, there exist consistent charge assignments. For $N = 4$, for example, one may take $(\phi_{\mathbf{10}}, \bar{\phi}_{\mathbf{5}}, \bar{\nu}, h, \bar{h}, \alpha) = (1, 1, 1, 0, 0, 1)$.

Among three possibilities, we are interested in the case of $(n_{\mathbf{5}'}, n_{\mathbf{10}'}) = (0, 1)$ because the newly introduced $\mathbf{10}$ multiplet may couple to the up-type Higgs boson if its Z_{NR} charge (denoted as $\phi'_{\mathbf{10}}$) is equal to that of $\Phi_{\mathbf{10}}$. Such a charge assignment does not conflict with any of the conditions because $\phi'_{\mathbf{10}}$ is arbitrary as far as Eq. (12) is satisfied. In the following, we concentrate on the case with an extra pair of $\mathbf{10} + \bar{\mathbf{10}}$ multiplet, and study the mass of the lightest Higgs boson in such a case.

3 Higgs Mass

Now we discuss the lightest Higgs mass, paying particular attention to the contributions of loop diagrams with extra matters inside the loop. As discussed in the previous section, some

^{#2}If SUSY invariant masses for them are allowed, their R charges should satisfy $\phi' + \bar{\phi}' = 2\alpha$. In this case they do not contribute to the anomalies.

of the fields contained in Φ'_{10} may couple to up-type Higgs H_u if it has proper Z_{NR} charge.^{#3} Then, if its Yukawa interaction is large, we expect a sizable correction to the lightest Higgs mass as in the case of the top and stop [9, 10, 11].

To study the Higgs mass with extra matters, we first decompose Φ'_{10} and $\bar{\Phi}'_{10}$ as $\Phi'_{10} = Q + U + E$ and $\bar{\Phi}'_{10} = \bar{Q} + \bar{U} + \bar{E}$, where $Q(\mathbf{3}, \mathbf{2}, 1/6)$, $U(\bar{\mathbf{3}}, \mathbf{1}, -2/3)$, $E(\mathbf{1}, \mathbf{1}, 1)$, $\bar{Q}(\bar{\mathbf{3}}, \mathbf{2}, -1/6)$, $\bar{U}(\mathbf{3}, \mathbf{1}, 2/3)$, and $\bar{E}(\mathbf{1}, \mathbf{1}, -1)$ are gauge eigenstates of the standard-model gauge group. (The gauge quantum numbers for $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$ are shown in the parenthesis.) Then, the relevant part of the superpotential is given by^{#4}

$$W = y_t t_R^c q_L H_u + y_U U Q H_u + M_U \bar{U} U + M_Q \bar{Q} Q, \quad (16)$$

and the soft SUSY breaking terms are^{#5}

$$\begin{aligned} \mathcal{L}_{\text{soft}} = & m_{\tilde{q}}^2 |\tilde{q}_L|^2 + m_{\tilde{t}}^2 |\tilde{t}_R^c|^2 + m_{\tilde{Q}}^2 |\tilde{Q}|^2 + m_{\tilde{\bar{Q}}}^2 |\tilde{\bar{Q}}|^2 + m_{\tilde{U}}^2 |\tilde{U}|^2 + m_{\tilde{\bar{U}}}^2 |\tilde{\bar{U}}|^2 \\ & + (y_t A_t \tilde{t}_R^c \tilde{q}_L H_u + y_U A_U \tilde{U} \tilde{Q} H_u + \text{h.c.}), \end{aligned} \quad (17)$$

where $q_L(\mathbf{3}, \mathbf{2}, 1/6)$ and $t_R^c(\bar{\mathbf{3}}, \mathbf{1}, -2/3)$ are standard-model quarks in the third generation, which contain left- and right-handed top (s)quarks, respectively. In addition, the “tilde” is for superparticles. (So, \tilde{Q} is the scalar component in the superfield Q , for example.)

We presume that the M_Q - and M_U -parameters are generated by the Giudice-Masiero mechanism, so the masses of extra matters are expected to be as heavy as the MSSM superparticles. Then assuming a little hierarchy between the electroweak scale and the masses of superparticles, which is suggested by the sparticle search experiments, we estimate the Higgs mass by using the effective field theory approach. Then, the relevant theory describing the energy scale above M_{SUSY} (which is taken to be the “typical” mass of superparticles) is the MSSM with extra matters, while the low-energy effective theory below M_{SUSY} is the standard model. Two theories should be matched at $\mu = M_{\text{SUSY}}$ (with μ being the renormalization scale).

In our analysis, we consider the case that only one light Higgs doublet, which we call the standard-model-like Higgs doublet H_{SM} , remains below M_{SUSY} , which is consistent with the assumption that the low-energy effective theory below M_{SUSY} is the standard model. The potential of H_{SM} is denoted as

$$V_{\text{SM}} = m_H^2 |H_{\text{SM}}|^2 + \frac{1}{2} \lambda |H_{\text{SM}}|^4. \quad (18)$$

^{#3}Because the extra matters couple to the Higgs boson, one should care about the oblique corrections (i.e., so-called S - and T -parameters) due to these fields. In the present case, the dominant contribution to the masses of these new particles is from the gauge-invariant operators and hence the oblique corrections become suppressed as these extra particles become heavy. The oblique corrections become small enough if the new particles are as heavy as ~ 1 TeV; for more detail, see [8].

^{#4}For simplicity, we neglect the effects of possible CP violating phases in the new interaction terms; parameters y_U , M_Q , M_U , and A_U are all taken to be real.

^{#5}For simplicity, we assume that the bi-linear SUSY breaking terms for extra matters are negligible.

For the calculation of the lightest Higgs mass, we need to know the coupling constant λ at M_{SUSY} ; once $\lambda(M_{\text{SUSY}})$ is known, Higgs mass is estimated as^{#6}

$$m_h^2 = \lambda(m_h)v^2, \quad (19)$$

where $v \simeq 246$ GeV is the vacuum expectation value of the standard-model-like Higgs boson. Notice that $\lambda(m_h)$ is related to $\lambda(M_{\text{SUSY}})$ by solving renormalization group equation in the framework of the standard model.^{#7}

In the present case, $\lambda(M_{\text{SUSY}})$ is given by

$$\lambda(M_{\text{SUSY}}) = \frac{1}{4}(g_2^2 + g_1^2) \cos^2 2\beta + \delta\lambda_{\tilde{t}} + \delta\lambda', \quad (20)$$

where g_2 and g_1 are gauge coupling constants for $SU(2)_L$ and $U(1)_Y$, respectively. In addition, $\delta\lambda_{\tilde{t}}$ is the threshold correction at M_{SUSY} due to the stop loop diagram, while $\delta\lambda'$ is that from diagrams with extra matters inside the loop. Taking $m_{\tilde{q}} = m_{\tilde{t}}$ for simplicity, we obtain [14]

$$\delta\lambda_{\tilde{t}} = \frac{3y_t^4 \sin^4 \beta}{8\pi^2} \left(\frac{A_t^2}{m_{\tilde{t}}^2} - \frac{A_t^4}{12m_{\tilde{t}}^4} \right). \quad (21)$$

We study the effect of the extra matters using one-loop contribution to the effective potential:

$$\Delta V = \Delta V^{(\text{B})} + \Delta V^{(\text{F})}, \quad (22)$$

where $\Delta V^{(\text{B})}$ and $\Delta V^{(\text{F})}$ are contributions of bosonic and fermionic loops, respectively. $\Delta V^{(\text{B})}$ is given by

$$\Delta V^{(\text{B})} = \frac{3}{32\pi^2} \text{Tr} \left[(\mathcal{M}_{\text{B}}^2 + \Delta\mathcal{M}_{\text{B}}^2)^2 \left\{ \ln \left(\frac{\mathcal{M}_{\text{B}}^2 + \Delta\mathcal{M}_{\text{B}}^2}{\mu^2} \right) - \frac{3}{2} \right\} \right], \quad (23)$$

where

$$\mathcal{M}_{\text{B}}^2 = \text{diag}(M_Q^2 + m_Q^2, M_Q^2 + m_{\tilde{Q}}^2, M_U^2 + m_U^2, M_U^2 + m_{\tilde{U}}^2), \quad (24)$$

and

$$\Delta\mathcal{M}_{\text{B}}^2 = \begin{pmatrix} y_U^2 |H_u|^2 & 0 & y_U A_U H_u^* & y_U M_U H_u^* \\ 0 & 0 & y_U M_Q H_u^* & 0 \\ y_U A_U H_u & y_U M_Q H_u & y_U^2 |H_u|^2 & 0 \\ y_U M_U H_u & 0 & 0 & 0 \end{pmatrix}. \quad (25)$$

^{#6}For the calculation of $\lambda(m_h)$, the most important effect below M_{SUSY} is from the Yukawa interaction with the top quark. Thus, for our renormalization group analysis, we stop the running of λ at $\mu = m_t$.

^{#7}In our numerical analysis, we use the top-quark mass of $m_t^{(\text{pole})} = 172.9$ GeV [12]. The pole mass is related to the $\overline{\text{MS}}$ mass as [13]

$$\frac{m_t^{(\text{pole})}}{m_t^{(\overline{\text{MS}})}(m_t^{(\text{pole})})} = 1 + \frac{4}{3} \frac{\alpha_s(m_t^{(\text{pole})})}{\pi}.$$

Furthermore,

$$\Delta V^{(F)} = - \Delta V^{(B)} \Big|_{A_U=m_Q^2=m_{\tilde{Q}}^2=m_{\tilde{U}}^2=m_{\tilde{c}}^2=0}. \quad (26)$$

Calculating the coefficient of $|H_u|^4$ term in ΔV and replacing $H_u \rightarrow H_{\text{SM}} \sin \beta$, $\delta\lambda'$ is obtained. Because Q and U are in a same multiplet of $SU(5)_{\text{GUT}}$, the relation $M_Q = M_U$ holds at the GUT scale. This equality is violated by the renormalization group effect below the GUT scale, but the most important effect, i.e., the QCD effect, does not spoil this relation. Thus, we adopt the approximation $M_Q = M_U$ (at $\mu = M_{\text{SUSY}}$). In addition, for simplicity, we approximate $m_{\tilde{Q}}^2 = m_{\tilde{q}}^2 = m_{\tilde{U}}^2 = m_{\tilde{c}}^2$. Then, $\delta\lambda'$ is given by

$$\begin{aligned} \delta\lambda' = & \frac{3y_U^4 \sin^4 \beta}{8\pi^2} \ln \left(\frac{M_U^2 + m_{\tilde{U}}^2}{M_U^2} \right) \\ & - \frac{y_U^4 \sin^4 \beta}{32\pi^2} \frac{A_U^4 - (8M_U^2 + 12m_{\tilde{U}}^2)A_U^2 + 8M_U^2 m_{\tilde{U}}^2 + 10m_{\tilde{U}}^4}{(M_U^2 + m_{\tilde{U}}^2)^2}. \end{aligned} \quad (27)$$

One can see that, with a relevant choice of parameters, $\delta\lambda'$ becomes positive and an enhancement of the lightest Higgs mass happens.

To see how large the Higgs mass can be, we calculate m_h . To make our discussion simple, we take

$$m_t^2 = m_{\tilde{q}}^2 = m_{\tilde{Q}}^2 = m_{\tilde{c}}^2 = m_{\tilde{U}}^2 = m_{\tilde{c}}^2 \equiv m_{\text{SUSY}}^2, \quad (28)$$

and the tri-linear coupling constants are parametrized as

$$A_t = a_t m_{\text{SUSY}}, \quad A_U = a_U m_{\text{SUSY}}. \quad (29)$$

For our numerical calculation, we take $M_{\text{SUSY}} = m_{\text{SUSY}}$.

In Fig. 1, we plot the lightest Higgs mass on y_U (at $\mu = M_{\text{SUSY}}$) vs. a_U plane for several values of $\tan \beta$, taking $m_{\text{SUSY}} = 1$ TeV, $M_U = M_Q = 1$ TeV, and $a_t = a_U$. If the Yukawa coupling constants are too large, they diverge below the GUT scale. Requiring the perturbativity (i.e., $y_U^2 \lesssim 4\pi$) below the GUT scale, the upper bound on $y_U(m_{\text{SUSY}})$ is obtained; in the figure, such a bound is also shown. In addition, as one can see from Eq. (21), $\delta\lambda_t$ takes its maximal value when $a_t = \sqrt{6}$; results for such a case are shown in Fig. 2.

One can see that the radiative correction due to the extra particles may drastically change the lightest Higgs mass; m_h can be significantly enhanced compared to the case of the MSSM [15, 16]. We note here that the lightest Higgs mass is sensitive to the A_U -parameter. In particular, when $a_U \sim 3$ and $y_U \sim 1$, the lightest Higgs mass becomes as heavy as ~ 140 GeV even if we assume the perturbativity of the Yukawa coupling constant up to the GUT scale. Such a value of m_h is above the MSSM bound on the lightest Higgs mass, which is ~ 125 GeV if $m_{\text{SUSY}} = 1$ TeV [17]. We have also checked that a larger value of m_h is also possible for a different value of M_U or $m_{\tilde{U}}$. As the ratio $m_{\tilde{U}}^2/M_U^2$ becomes large, the logarithmic term in Eq. (27) is enhanced, resulting in a larger value of m_h . For example, when $M_U = 500 - 750$ GeV (and $m_{\text{SUSY}} = 1$ TeV), m_h can be made as large as 145 – 150 GeV.

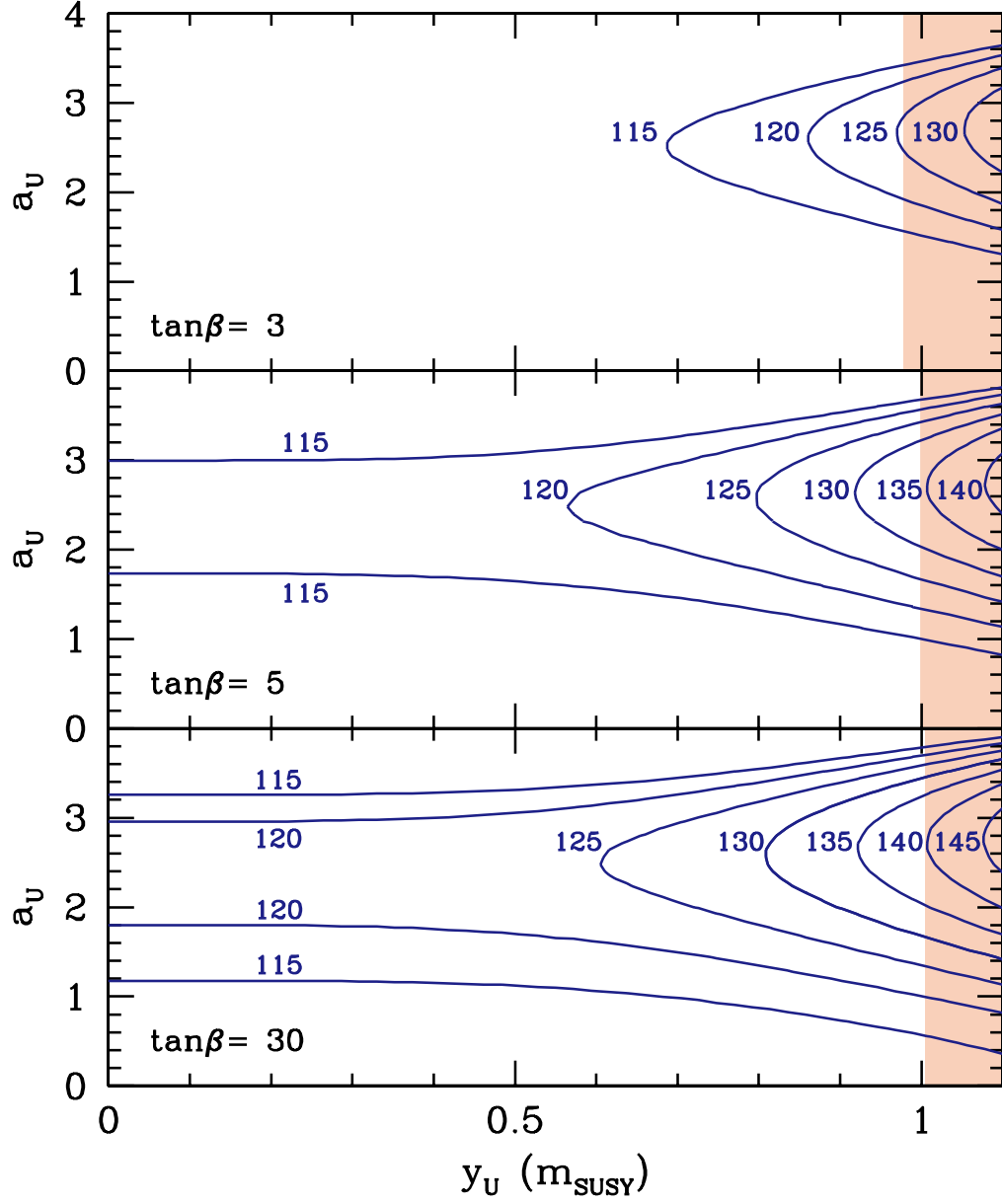


Figure 1: Contours of constant m_h on y_U vs. a_U plane for $\tan\beta = 3, 5$, and 30 . Here, we have taken $m_{\text{SUSY}} = M_U = M_Q = 1$ TeV, and $a_t = a_U$. In the shaded region, y_U becomes non-perturbative below the GUT scale. Numbers in the figure are the lightest Higgs mass in units of GeV.

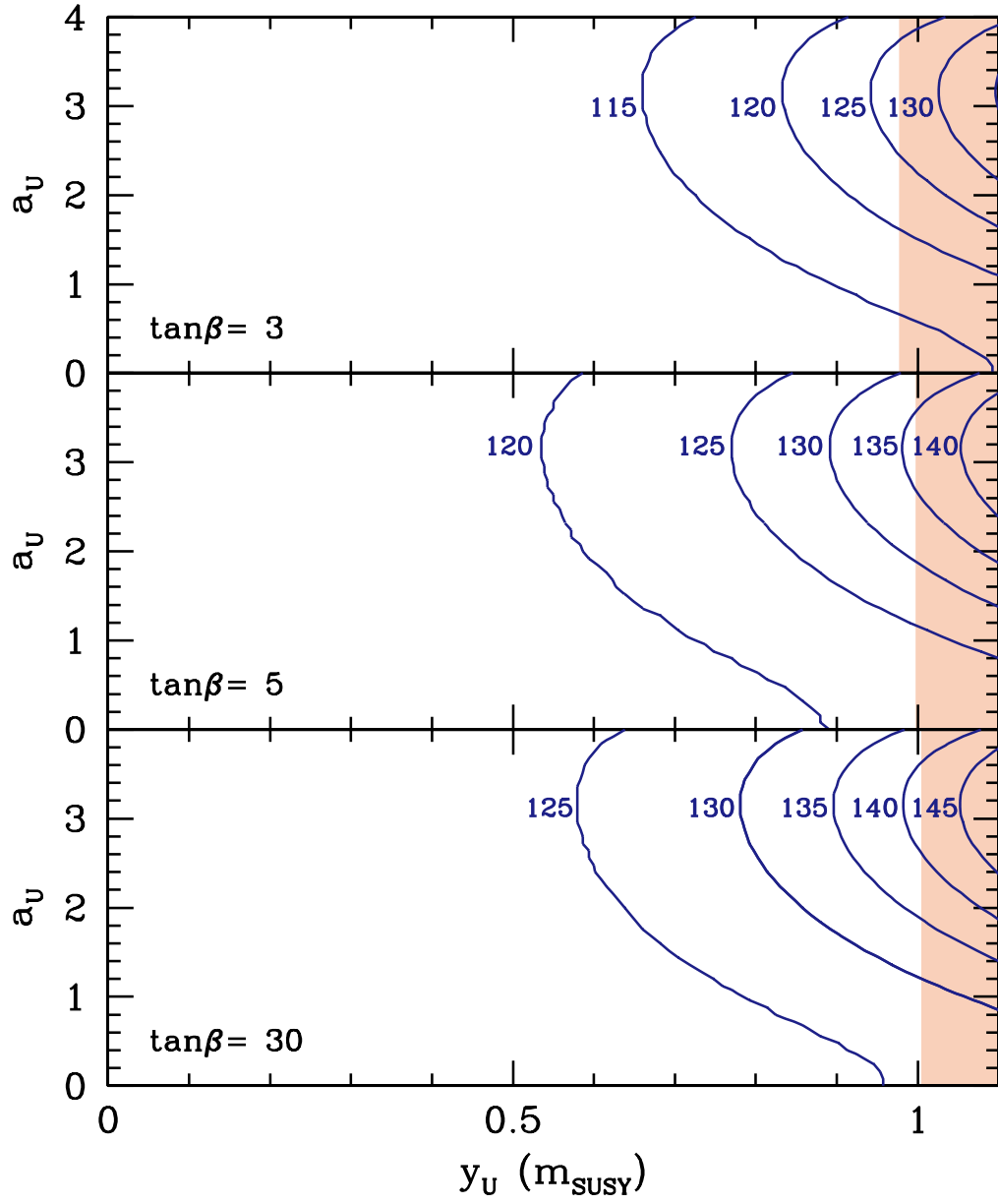


Figure 2: Same as Fig. 1, except for $a_t = \sqrt{6}$.

4 Discussion

In this paper, we have discussed the lightest Higgs mass in a model with a non-anomalous discrete R -symmetry. For the cancellation of the gauge anomaly, extra particles should be added to the MSSM; we have seen that the gauge anomaly can be cancelled out by adding $\mathbf{10} + \bar{\mathbf{10}}$ multiplet of $SU(5)_{\text{GUT}}$. In such a model, the SUSY invariant mass term arises for the $\mathbf{10} + \bar{\mathbf{10}}$ multiplet via the Giudice-Masiero mechanism, and the particles in the $\mathbf{10} + \bar{\mathbf{10}}$ multiplet becomes as light as MSSM superparticles. We have paid particular attention to the lightest Higgs mass in such a model, and we have seen that m_h can become as large as ~ 140 GeV (or larger).

This fact has a great impact on the study of SUSY models because the Higgs mass is the crucial check point of low-energy SUSY and also because the LHC experiment is expected to find Higgs boson in near future. We have shown that the significant enhancement of the Higgs mass is possible if extra particles from $\mathbf{10}$ multiplet of $SU(5)_{\text{GUT}}$ exist; such a modification is well-motivated to realize a non-anomalous discrete R -symmetry. In particular, the ATLAS group recently observed $\sim 2.8\sigma$ excess of the Higgs-like events in the mass range of $\sim 120 - 140$ GeV [18]. If the existence of the Higgs boson heavier than the MSSM bound is confirmed, it is strongly suggested to look for extra particles in $\mathbf{10} + \bar{\mathbf{10}}$ multiplet.

Our estimation of the Higgs mass is based on the renormalization-group analysis with taking account of the leading-order threshold correction at $\mu = M_{\text{SUSY}}$; the sub-dominant contributions are expected to be suppressed by powers of v/M_{SUSY} or v/M_U . Such sub-dominant contributions may slightly change the lightest Higgs mass. For more precise determination of m_h , the full one-loop calculation of the effective potential is needed, which is beyond the scope of this paper. However, in order to estimate the accuracy of our results, we have compared our results (for the case without extra matter) with those of FeynHiggs package [19, 20, 21, 22] which is expected perform a precise calculation of the Higgs mass in the framework of the MSSM. We found that the difference between two results are within ~ 5 GeV.

Before closing this paper, several comments are in order. First comment is on the stability of extra particles. If we strictly adopt the superpotential given in Eq. (16) and soft SUSY breaking terms given in Eq. (17), the lightest extra particle becomes stable. If a charged or colored particle becomes stable, it may conflict with cosmological constraints. However, the extra particles can decay into standard-model quarks or leptons (and weak boson) if they slightly mix with standard-model particles. Because the Z_{NR} charges of the extra particles are same as or opposite to that of the standard-model fermions, such mixing naturally exists.

We have seen that the enhancement of the lightest Higgs mass becomes significant when $\tan\beta$ is large. This fact has an advantage if we take the muon $(g-2)$ anomaly seriously. If we compare the face values of the measured value of the muon anomalous magnetic moment with the theoretical prediction, they have 3.3σ discrepancy [23]. In low-energy supersymmetric models, SUSY contribution to the muon anomalous magnetic moment becomes sizable in particular when $\tan\beta$ is large [24, 25], which may be the origin of the muon $(g-2)$ anomaly. In the present set up, thus the muon $(g-2)$ anomaly may be solved with realizing the

Higgs mass much larger than the MSSM upper bound. This is a big contrast to the case with a singlet Higgs, i.e., the so-called the next to the MSSM (NMSSM), which may also enhance the lightest Higgs mass; in the NMSSM, $\tan\beta$ is required to be relatively small for the enhancement of m_h [26].

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